

AD-A116 430 MASSACHUSETTS INST OF TECH CAMBRIDGE RESEARCH LAB OF--ETC F/G 12/1  
MULTI-DIMENSIONAL SPECTRAL ESTIMATION FOR UNEQUALLY SPACED DATA--ETC(U)  
JUN 82 J H MCCLELLAN DAAG29-81-K-0073

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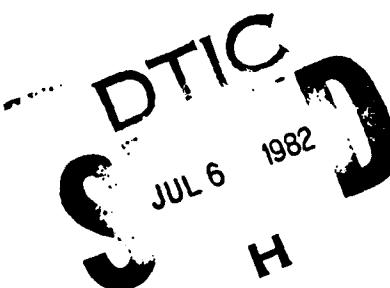
FINAL REPORT

U. S. Army Research Office  
Contract DAAG 29-81-K-0073

covering the period  
April 1, 1981 - June 30, 1982

Submitted by  
J. H. McClellan

June 28, 1982



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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
		AD-A116 430
4. TITLE (and Subtitle) MULTI-DIMENSIONAL SPECTRAL ESTIMATION FOR UNEQUALLY SPACED DATA	5. TYPE OF REPORT & PERIOD COVERED Progress Report Apr. 13, 1981 - Jun. 30, 1982	
7. AUTHOR(s) J. H. McClellan	6. PERFORMING ORG. REPORT NUMBER DAAG 29-81-K-0073	
9. PERFORMING ORGANIZATION NAME AND ADDRESS Research Laboratory of Electronics Massachusetts Institute of Technology Cambridge, Massachusetts 02139	10. PROGRAM, ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS P-18030-EL	
11. CONTROLLING OFFICE NAME AND ADDRESS U. S. Army Research Office P. O. Box 12211 Research Triangle Park, NC 27709	12. REPORT DATE June 1982	
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	13. NUMBER OF PAGES 4	
15. SECURITY CLASS. (of this report) Unclassified		
15a. DECLASSIFICATION/DOWNGRADING SCHEDULE		
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES The view, opinions, and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Pisarenko's method of spectral estimation Multi-dimensional spectral estimation technique		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) Research has been carried out in two areas: first, the extension of Pisarenko's method to multidimensional (m-D) signals and arrays; second, the extension of the maximum entropy method (MEM) to non-uniformly sampled m-D signals and arrays. Most of this work has been of a theoretical nature.		

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**Multi-Dimensional Spectral Estimation for Unequally Spaced Data**  
**PI: James H. McClellan**

Research has been carried out in two areas: first, the extension of Pisarenko's method to multidimensional ( $m$ -D) signals and arrays; second, the extension of the maximum entropy method (MEM) to non-uniformly sampled  $m$ -D signals and arrays. Most of this work has been of a theoretical nature.

A procedure for calculating Pisarenko's estimate has been derived. The form of the estimate is that of sinusoids (or plane waves) in a background of noise with known spectral content, but unknown total power. The problem can be reduced to a linear program that is computationally similar to the one used for designing two-dimensional finite impulse response (FIR) digital filters.

In the process of deriving Pisarenko's spectral estimate, one is led naturally to consider the "extendibility" problem: Given a finite set of samples of a presumed autocorrelation function (acf), does there exist a positive power spectrum that exactly matches the given acf samples? This extendibility problem is important in practice. Often the autocorrelation function is estimated from data samples and it is necessary to have a test to determine whether such acf estimates are extendible. One possible test is to compute Pisarenko's estimate under the assumption that the background noise is white. In the course of the computation, the white noise power will be determined; if it turns out to be negative, the acf estimates are not extendible, if positive they are extendible. If the noise power were zero, then the power spectrum would consist only of impulses. Improvements on this test will be needed in the future.

The papers by Lang and McClellan [1,2,3] develop the theory of Pisarenko's method in detail. Connections to some practical array processing problems are also given. The review papers [4,5] by McClellan also discuss Pisarenko's method in the context of the general multidimensional spectral estimation problem.

The second major topic of research was the development of a general multidimensional MEM spectral estimation algorithm. A number of such algorithms have been proposed, but the research under this contract [6,7,8,9] is unique in that a rigorous mathematical approach was used to exploit fully the convex nature of the MEM optimization problem. In particular, a general necessary and sufficient condition for the existence of the  $m$ -D MEM spectral estimate has been obtained. This existence condition comes about when a dual entropy minimization problem is derived. The dual problem is particularly important because it is a finite-dimensional convex optimization problem; whereas, the original MEM problem is infinite-dimensional. Much of the previous work on MEM used some form of the dual problem, but not the

form derived in this work. A most important point is the convex nature of the problem which guarantees that traditional numerical optimization methods such as steepest descent or quasi-Newton will converge.

A final area of investigation is the use of the new m-D spectral estimation techniques in a practical setting. Some work [10,11] has been published on the comparison of MEM with interpolation for bearing estimation with non-uniform arrays. This research has shown the superiority of the general MEM algorithm for synthetic signals, consisting of sinusoids in noise. Work is presently underway to apply these algorithms to data from seismic arrays.

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